Mean Velocity Profile of a Thick Turbulent Boundary Layer along a Circular Cylinder

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Theme

COMPETING hypotheses for the mean velocity profile of a turbulent boundary layer on a cylinder in axial flow are discussed and their results explicitly compared with experiment.

Content

The mean velocity profile of a turbulent boundary layer on a cylinder in axial flow without pressure gradient may be written, over some inner range of wall distances y, in the functional form

$$u/v_* = F(v_* y/v, y/a) \tag{1}$$

where $u(x, y)$ denotes mean velocity, $v_*(x)$ friction velocity, v kinematic viscosity, and a cylinder radius. Within the viscous sublayer, apart from terms of order (y/a),[4] the explicit result for a laminar cylindrical boundary layer can be shown to apply and written as

$$u/v_* = (av_*/v)\ln(1+y/a) \tag{2}$$

Hypotheses have been proposed previously that yield the function F in Eq. (1) for a cylinder in terms of its limiting form, $F(v_* y/v, 0)$, for a planar layer:

1) Coles' streamline hypothesis

$$F(y_+, \Delta) = F[y_+(1+\Delta/2), 0] \tag{3}$$

2) Rao hypothesis

$$F(y_+, \Delta) = F[(y_+/\Delta)\ln(1+\Delta), 0] \tag{4}$$

3) Derivative hypothesis

$$\partial F(y_+, \Delta)/\partial y_+ = (1+\Delta)^{-1} dF(y_+, 0)/dy \tag{5}$$

As already known, the streamline hypothesis fails even to conform (to the requisite order) to the known result Eq. (2).[1-3]

If $\partial u/\partial y$ is related to shear stress via eddy viscosity, $\varepsilon(x, y)$, in the usual way and the shear force per unit streamwise distance is independent of y, the hypotheses may be expressed in terms of ε. The derivative hypothesis corresponds to an eddy viscosity at given y and v_* that is independent of cylinder radius. Where $F(y_+, 0) = A\ln y_+ + B$ in the right members of Eq. (3–5), in particular, the derivative Rao, and streamline hypotheses yield, respectively, $\varepsilon = A^{-1}v_* y$, $A^{-1}v_* a\ln(1+y/a)$, $A^{-1}v_* y(1+y/2a)/(1+y/a)$.[2] According to the Rao form, the mixing length or size of large eddies responsible for eddy viscosity increases as wall distance only for $y \lesssim a$ and beyond this only logarithmically, somewhat as suggested by wall-pressure measurements.

The Rao profile for the logarithmic layer can be derived by an argument exactly analogous to the standard one adduced by Millikan for a planar boundary layer.[4] Generalized to include the outer layer, Eq. (1) may be written as

$$u/v_* = f(v_* \delta/v, y/\delta, \delta/a) \tag{6}$$

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In some inner range that, by Eq. (2), includes at least the viscous sublayer this function has the form $u/v_* = \phi[(v_* a/v)\ln(1+y/a)]$. Suppose now that in the outer, wake-like part of the layer a velocity-defect law holds analogous to that in planar flow: $(U-u)/v_* = \psi(y/\delta, \delta/a)$, where U is the freestream velocity. On assumption of a common range, we find a form in the range of overlap given by $\phi[a_+\ln(1+y/a)] = A\ln[a_+\ln(1+y/a)]+B$, where A and B are constant.

The competing Rao and derivative hypotheses (4) and (5) can be generalized by reference to f in Eq. (6) to include the outer layer.

We have computed profiles from the alternative hypotheses (3–5) by use of Squire's transitional profile for the planar limit, given by

$$F(y_+, 0) = \begin{cases} y_+ & \text{for } 0 < y_+ < c_1, \\ A\ln(y_+ - J_1) + B_1 & \text{for } c_1 < y_+ \end{cases} \tag{7}$$

with $A = 2.5$, $B = 5.1$, and $c_1 = A\ln A + B_1 = 7.4$, $J_1 = c_1 - A = 4.9$. (This corresponds to $\varepsilon = A^{-1}v_*(y - c_1 v/v_*)$.) Results were compared with two profiles measured by Richmond[5] on a model of radius $a = 0.012$ in., namely the first and fourth in Table 2 of this reference, corresponding, respectively, to $U = 1240$ cm/sec with natural transition and $U = 460$ fps with clay-centerbody trip and eveloping stovepipe for flow alignment.

Since the friction velocity v_* was not measured and not reliably derivable from the slope of the measured profile at small wall distances, this parameter was regarded as adjustable for each of hypotheses (3–5) and was adjusted in two different ways for each set. In the first instance (yielding the more clear-cut comparison) the values of v_* were chosen to yield a common value of $u(y)$ in agreement with the measured result at a chosen y well out in the boundary layer (but at a minor fraction of boundary-layer thickness). In the second instance the values were chosen instead to yield a common value in agreement with the measured $u(y)$ at the measured point nearest the wall. The respective values of a_+ (whence follows v_*) implied by the three hypotheses (and by a planar layer, for reference) are given in Table 1 for the two types of fit for $U = 1240$ cm/sec. Computed and measured profiles are given in Figs. 1 and 2; along with absolute scales in Fig. 1, scales for u/v_* vs $10\log(v_* y/v)$ are given where the value of v_* is that for the Rao profile.

As illustrated, there is no possible value of v_* for which either the streamline or derivative hypothesis yields a reasonable fit over as much as the inner tenth of the boundary-layer thickness. In contrast, the Rao hypothesis succeeds in describing the profile over most of the thickness, even at very large y/a. These comparisons provide a clear demonstration supporting previous con-

Table 1 Values of parameter av_*/v used for profiles in Figs. 1 and 2

Hypothesis	Fig. 1	Fig. 2
Streamline (0)	8.92	12.70
(Planar) (3)	11.9	14.20
Rao (1)	16.2	15.68
Derivative (2)	21.6	15.74

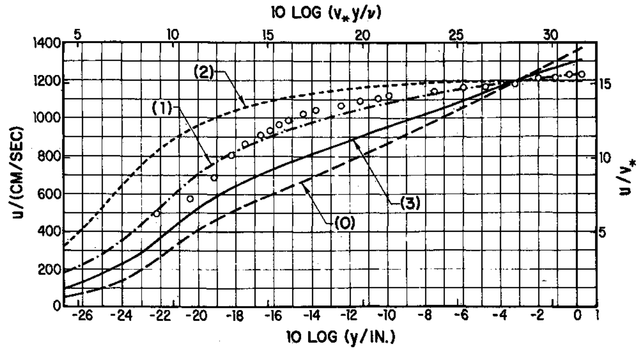

Fig. 1 Measured mean velocity profile for cylinder[5] and those computed from various hypotheses with friction velocities chosen to yield measured velocity at $y = 0.477$ in. Dimensionless scales refer to $v_* = 78.9$ cm/sec of Rao curve.

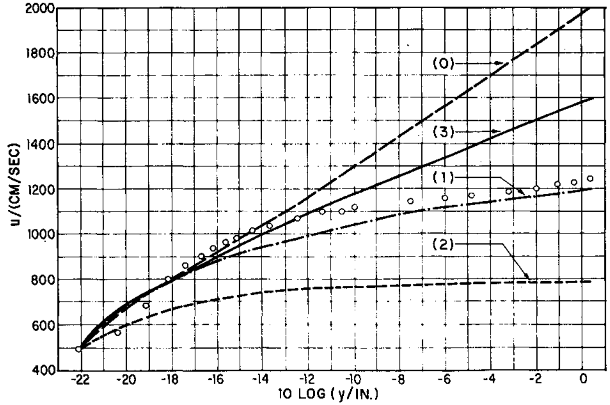

Fig. 2 Mean velocity profiles as in Fig. 1, but with friction velocities chosen to yield measured velocity at $y = 0.006$ in.

clusions.[1–3] The advantage of the Rao hypothesis is not substantially weakened if the result is compared with that[3] of assuming an eddy viscosity independent of y/a, as in the derivative hypothesis, but integrating the mean momentum and continuity equations instead of assuming shear force per unit streamwise distance independent of y.

Turbulent-boundary-layer growth on a cylinder where $\delta/a \gg 1$ and on a plane may be compared in the crude approximation of constant friction coefficients.† The momentum thickness δ_m for the cylinder is then $(v_*/U)(2ax)^{1/2}$, and that for the plane $(v_*/U)x$. The thickness δ over which $U - u \gtrsim v_*$ can be estimated for the cylinder as $\delta \sim [(v_*/U)^2 ax^2]^{1/3}$, whereas for the plane $\delta \sim (v_*/U)x$.

† Mean cylindrical boundary-layer properties have been calculated from the Rao hypothesis by White.[2] A term has been omitted, however, with unassessed effect in Eq. (13) and subsequent equations.

References

[1] Rao, G. N. V., "The Law of the Wall in a Thick Axisymmetric Turbulent Boundary Layer," *Journal of Applied Mechanics, Transactions of the ASME*, Vol. 34, 1967, pp. 237–238.

[2] White, F. M., "The Axisymmetric Turbulent Boundary Layer on an Extremely Long Cylinder," Rept. 1058, Nov. 1969, Navy Underwater Sound Lab., New London, Conn.; also ASME Paper 71-FE-25, *Journal of Basic Engineering*, to be published.

[3] Cebici, T., et al., "Laminar and Turbulent Incompressible Boundary Layers on Slender Bodies of Revolution in Axial Flow," *Journal of Basic Engineering, Transactions of the ASME*, 1970, pp. 545–554.

[4] Hinze, J. O., *Turbulence*, McGraw-Hill, New York, 1959, p. 466.

[5] Richmond, R. L., "Experimental Investigation of Thick Axially Symmetric Boundary Layers on Cylinders at Subsonic and Hypersonic Speeds," Hypersonic Research Project Memo No. 39, June 1957, Guggenheim Aeronautical Lab., California Institute of Technology, Pasadena, Calif.